Magnetic field extrapolation in close binary stars

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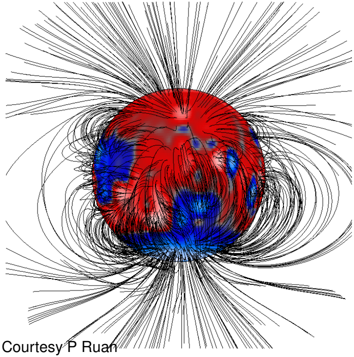
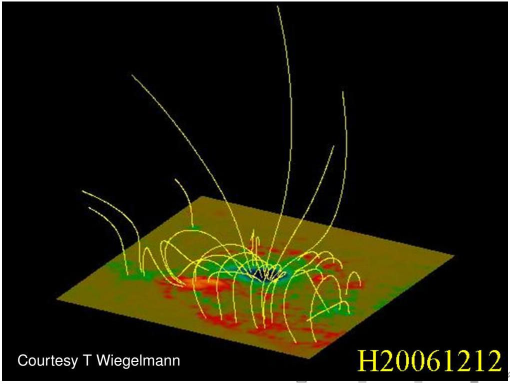
Solar magnetic fields
Coronal field extrapolation

- observation of coronal magnetic field hardly possible

⇒ approximation techniques:
  - **current-free** (potential) field approx.
  - **non-/linear force-free** field approx.
  - **magneto-hydrostatic** stratification



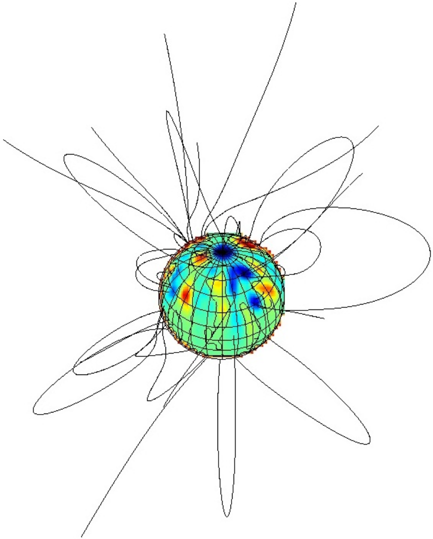
Courtesy P Ruan

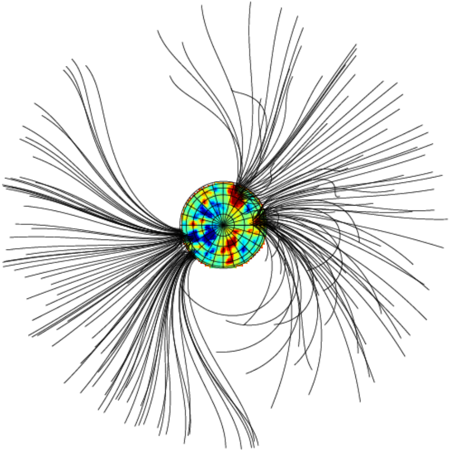


Courtesy T Wiegelmann

AB Dor

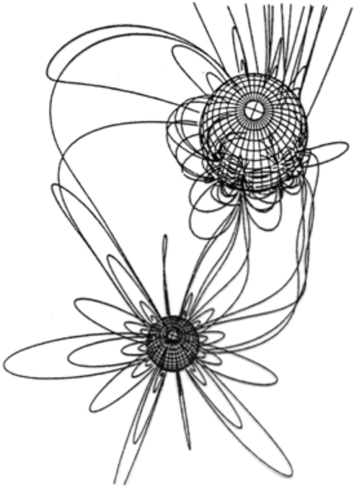
Potential field source surface (PFSS) approximation of coronal field structure (Jardine et al. 2002)





Joint magnetospheres of close star systems

Project overview



Uchida & Sakurai 1985

- targets:
  - e.g.: RS CVn-/BY Dra-systems
  - star-exoplanet systems
  - circumbinary exoplanets
  - hot binary stars
  - ...
- objectives:
  - structure of joint magnetosphere
  - open/closed/inter-connecting magnetic flux
  - magnetic interactions
  - atmospheric structure
  - observational signatures
  - ...

- Assumption: **current-free** magnetic field

$$\nabla \times \mathbf{B} = 0 \quad \rightarrow \quad \mathbf{B} = -\nabla \Psi \qquad \Psi\text{: scalar flux function}$$

- Solenoidal field:

$$\nabla \cdot \mathbf{B} = 0 \quad \rightarrow \quad \Delta \Psi = 0$$

→ **Laplace equation**

- Assumption: **current-free** magnetic field

$$\nabla \times \mathbf{B} = 0 \qquad \rightarrow \qquad \mathbf{B} = -\nabla\Psi \qquad \Psi\text{: scalar flux function}$$

- Solenoidal field:

$$\nabla \cdot \mathbf{B} = 0 \qquad \rightarrow \qquad \Delta\Psi = 0$$

$\rightarrow$ **Laplace equation**

- General solution:

$$\Psi(\mathbf{r}) = \sum_{l,m} a_{lm} R_{lm}(\mathbf{r}) + b_{lm} I_{lm}(\mathbf{r})$$

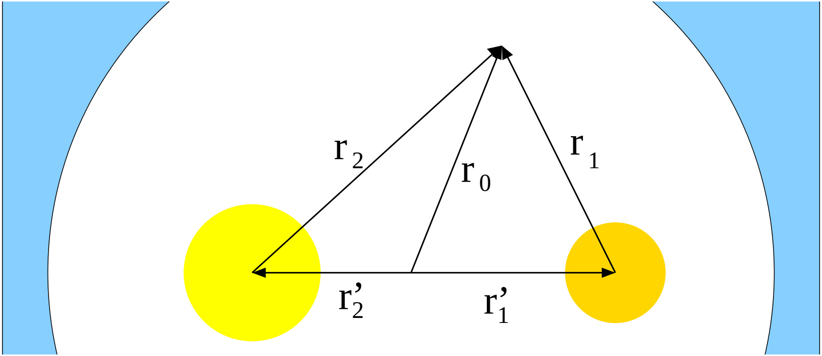
a_{lm}, b_{lm} expansion coefficients; Y_{lm} spherical harmonics

$$R_{lm}(\mathbf{r}) = r^l Y_{lm}(\theta, \phi) \qquad \textbf{regular} \text{ solid spherical harmonics}$$

$$I_{lm}(\mathbf{r}) = \frac{1}{r^{l+1}} Y_{lm}(\theta, \phi) \qquad \textbf{irregular} \text{ solid spherical harmonics}$$

Application to binary systems

Superposition of flux functions



$$\Psi(\mathbf{r}) = \Psi^{(0)}(\mathbf{r}_0) + \Psi^{(1)}(\mathbf{r}_1) + \Psi^{(2)}(\mathbf{r}_2)$$

- $\Psi^{(1)}(\mathbf{r}_1) = \mathbf{C}^{(1)} \cdot \mathbf{I}(\mathbf{r}_1)$ 'sources' inside primary
- $\Psi^{(2)}(\mathbf{r}_2) = \mathbf{C}^{(2)} \cdot \mathbf{I}(\mathbf{r}_2)$ 'sources' inside secondary
- $\Psi^{(0)}(\mathbf{r}_0) = \mathbf{C}^{(0)} \cdot \mathbf{R}(\mathbf{r}_0)$ 'mirror sources' outside source surface

- *Boundary conditions*:
  - Source surface: Stellar winds drag magnetic field in radial direction

$$\Psi\Big|_{S_0} = \Psi^{(0)}\Big|_{S_0}(\mathbf{r}_0) + \Psi^{(1)}\Big|_{S_0}(\mathbf{r}_1) + \Psi^{(2)}\Big|_{S_0}(\mathbf{r}_2) = \text{const.}$$

  - Primary and secondary: observed radial magnetic field maps

$$\begin{aligned} B_r^{(1)} &= -\frac{\partial}{\partial r_1}\left(\Psi^{(0)}(\mathbf{r}_0) + \Psi^{(1)}(\mathbf{r}_1) + \Psi^{(2)}(\mathbf{r}_2)\right)\Big|_{S_1} \\ B_r^{(2)} &= -\frac{\partial}{\partial r_2}\left(\Psi^{(0)}(\mathbf{r}_0) + \Psi^{(1)}(\mathbf{r}_1) + \Psi^{(2)}(\mathbf{r}_2)\right)\Big|_{S_2} \end{aligned}$$

Application to binary systems

Boundary conditions

- *Boundary conditions*:
  - Source surface: Stellar winds drag magnetic field in radial direction

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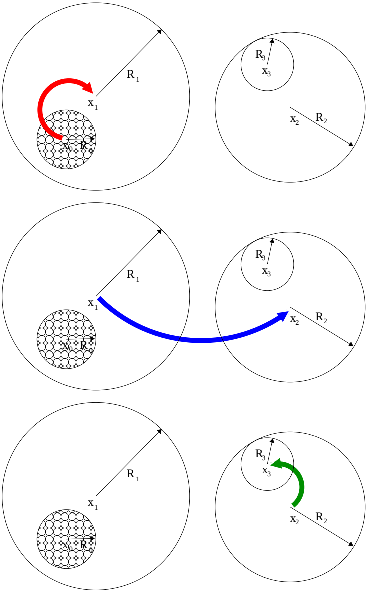
- *Problem*: Express $\Psi^{(p)}$ in reference frame of sphere q
- *Solution*: **Multipole translation theorems**

Multipole translation theorems



Irregular-irregular expansion: $|\mathbf{x}-\mathbf{x}_1| > |\mathbf{x}_1-\mathbf{x}_0|$

$$I_{lm}(\mathbf{x}-\mathbf{x}_0) = \sum_{l',m'} (I|I)_{lm}^{l'm'}(\mathbf{x}_1-\mathbf{x}_0)\, I_{l'm'}(\mathbf{x}-\mathbf{x}_1)$$

$$(I|I)_{lm}^{l'm'}(\mathbf{x}_1-\mathbf{x}_0) = (-1)^{l'+m'-l-m} R_{l'-l}^{m-m'}(\mathbf{x}_1-\mathbf{x}_0)$$

Irregular-regular expansion: $|\mathbf{x}-\mathbf{x}_2| < |\mathbf{x}_2-\mathbf{x}_1|$

$$I_{lm}(\mathbf{x}-\mathbf{x}_1) = \sum_{l'm'} (I|R)_{lm}^{l'm'}(\mathbf{x}_2-\mathbf{x}_1)\, R_{l'm'}(\mathbf{x}-\mathbf{x}_2)$$

$$(I|R)_{lm}^{l'm'}(\mathbf{x}_2-\mathbf{x}_1) = (-1)^{l'-m'} I_{l+l'}^{m-m'}(\mathbf{x}_2-\mathbf{x}_1)$$

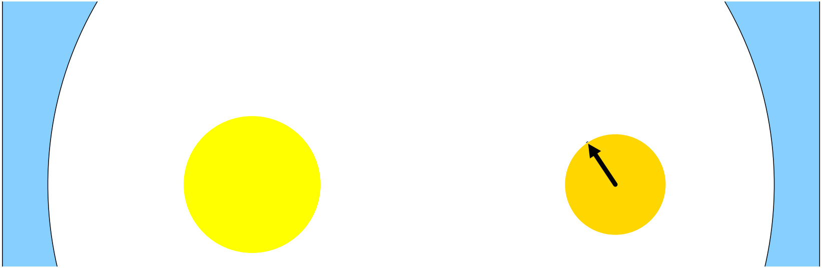
Regular-regular expansion:

$$R_{lm}(\mathbf{x}-\mathbf{x}_2) = \sum_{l',m'} (R|R)_{lm}^{l'm'}(\mathbf{x}_3-\mathbf{x}_2)\, R_{l'm'}(\mathbf{x}-\mathbf{x}_3)$$

$$(R|R)_{lm}^{l'm'}(\mathbf{x}_3-\mathbf{x}_2) = R_{l-l'}^{m-m'}(\mathbf{x}_3-\mathbf{x}_2)$$

Application to binary systems

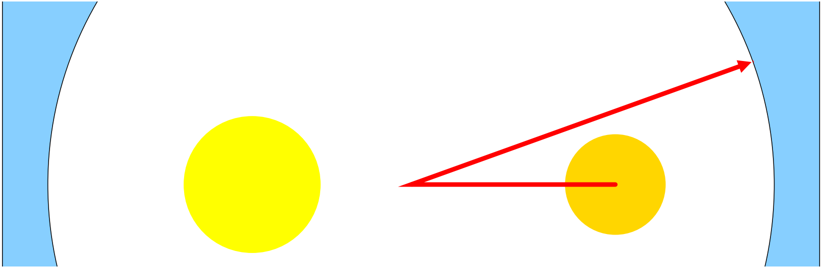
Example: Primary magnetic field



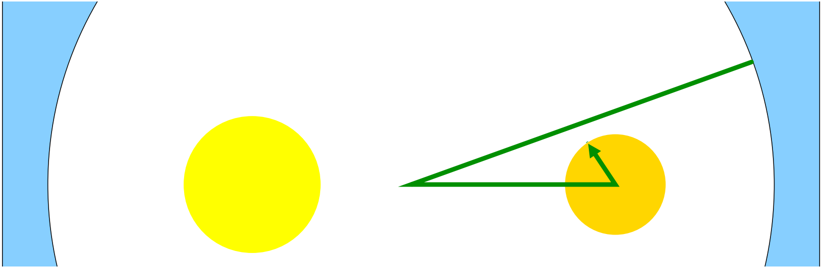
$$\mathbf{B}^{(1)} = -\mathbf{C}^{(1)} \cdot \left[\mathcal{I}'_{a_1} - (I|I)^{(10)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}^{-1}_{a_0} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1} \right]$$
$$- \mathbf{C}^{(2)} \cdot \left[(I|R)^{(21)} \cdot \mathcal{R}'_{a_1} - (I|I)^{(20)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}^{-1}_{a_0} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1} \right]$$

Application to binary systems

Example: Primary magnetic field



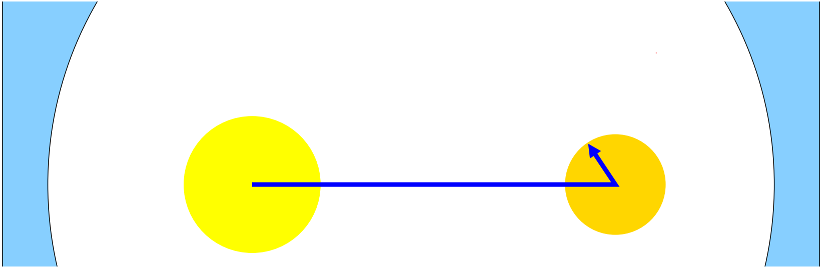
$$\mathbf{B}^{(1)} = -\mathbf{C}^{(1)} \cdot \left[\mathcal{I}'_{a_1} - (I|I)^{(10)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}^{-1}_{a_0} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1}\right] - \mathbf{C}^{(2)} \cdot \left[(I|R)^{(21)} \cdot \mathcal{R}'_{a_1} - (I|I)^{(20)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}^{-1}_{a_0} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1}\right]$$



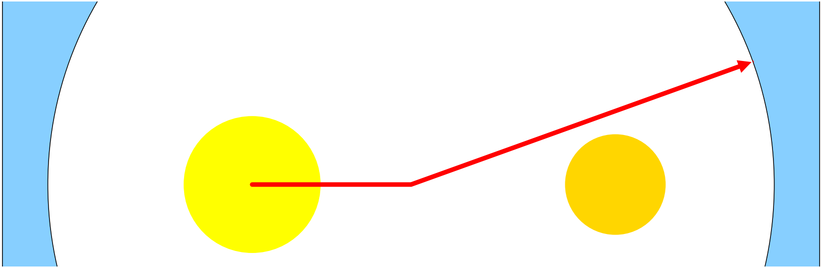
$$\mathbf{B}^{(1)} = -\mathbf{C}^{(1)} \cdot \left[\mathcal{I}'_{a_1} - (I|I)^{(10)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}_{a_0}^{-1} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1} \right] - \mathbf{C}^{(2)} \cdot \left[(I|R)^{(21)} \cdot \mathcal{R}'_{a_1} - (I|I)^{(20)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}_{a_0}^{-1} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1} \right]$$

Application to binary systems

Example: Primary magnetic field



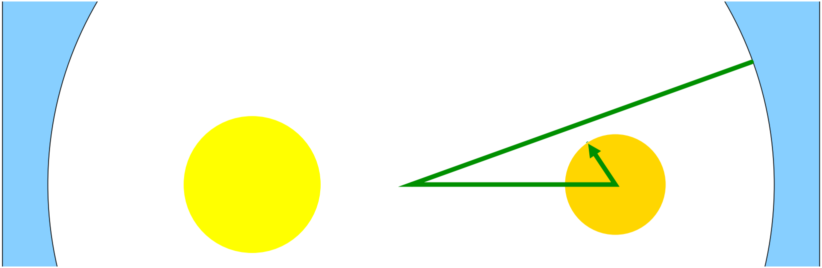
$$\mathbf{B}^{(1)} = -\mathbf{C}^{(1)} \cdot \left[\mathcal{I}'_{a_1} - (I|I)^{(10)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}_{a_0}^{-1} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1} \right] - \mathbf{C}^{(2)} \cdot \left[(I|R)^{(21)} \cdot \mathcal{R}'_{a_1} - (I|I)^{(20)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}_{a_0}^{-1} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1} \right]$$



$$\mathbf{B}^{(1)} = -\mathbf{C}^{(1)} \cdot \left[\mathcal{I}'_{a_1} - (I|I)^{(10)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}^{-1}_{a_0} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1} \right] - \mathbf{C}^{(2)} \cdot \left[(I|R)^{(21)} \cdot \mathcal{R}'_{a_1} - (I|I)^{(20)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}^{-1}_{a_0} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1} \right]$$

Application to binary systems

Example: Primary magnetic field



$$\mathbf{B}^{(1)} = -\mathbf{C}^{(1)} \cdot \Big[\mathcal{I}'_{a_1} - (I|I)^{(10)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}^{-1}_{a_0} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1} \Big] - \mathbf{C}^{(2)} \cdot \Big[(I|R)^{(21)} \cdot \mathcal{R}'_{a_1} - (I|I)^{(20)} \cdot \mathcal{I}_{a_0} \cdot \mathcal{R}^{-1}_{a_0} \cdot (R|R)^{(01)} \cdot \mathcal{R}'_{a_1} \Big]$$

Application to binary systems

Linear system of equations

- binary system:

$$\left(\begin{array}{c} \mathbf{B}^{(1)} \\ \mathbf{B}^{(2)} \end{array} \right) = \left(\begin{array}{cc} \mathcal{D}^{(11)} & \mathcal{D}^{(12)} \\ \mathcal{D}^{(21)} & \mathcal{D}^{(22)} \end{array} \right) \cdot \left(\begin{array}{c} \mathbf{C}^{(1)} \\ \mathbf{C}^{(2)} \end{array} \right)$$

- magnetic maps $\mathbf{B}^{(1/2)}$ known → solve for coefficients $\mathbf{C}^{(1/2)}$

- binary system:

$$\begin{pmatrix} \mathbf{B}^{(1)} \\ \mathbf{B}^{(2)} \end{pmatrix} = \begin{pmatrix} \mathcal{D}^{(11)} & \mathcal{D}^{(12)} \\ \mathcal{D}^{(21)} & \mathcal{D}^{(22)} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{C}^{(1)} \\ \mathbf{C}^{(2)} \end{pmatrix}$$

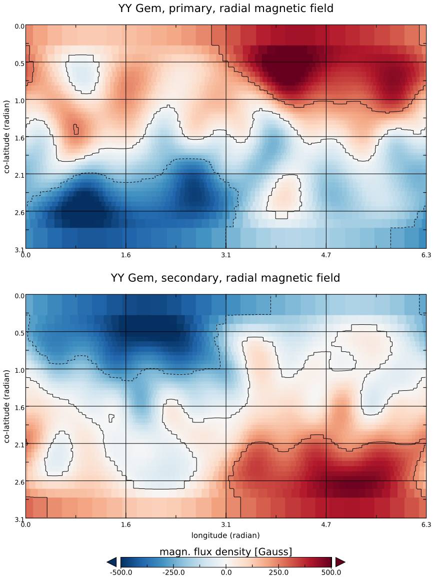
- magnetic maps $\mathbf{B}^{(1/2)}$ known → solve for coefficients $\mathbf{C}^{(1/2)}$

- *post scriptum*: technique easily expandable to N objects

$$\begin{pmatrix} \mathbf{B}^{(1)} \\ \vdots \\ \mathbf{B}^{(j)} \\ \vdots \\ \mathbf{B}^{(N)} \end{pmatrix} = \begin{pmatrix} \mathcal{D}^{(11)} & \dots & \mathcal{D}^{(1j)} & \dots & \mathcal{D}^{(1N)} \\ \vdots & \ddots & \vdots & & \vdots \\ \mathcal{D}^{(j1)} & \dots & \mathcal{D}^{(jj)} & \dots & \mathcal{D}^{(jN)} \\ \vdots & & \vdots & \ddots & \vdots \\ \mathcal{D}^{(N1)} & \dots & \mathcal{D}^{(Nj)} & \dots & \mathcal{D}^{(NN)} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{C}^{(1)} \\ \vdots \\ \mathbf{C}^{(j)} \\ \vdots \\ \mathbf{C}^{(N)} \end{pmatrix}$$

Example: YY Gem

Magnetic surface maps (Donati et al. 2011)



Example: YY Gem

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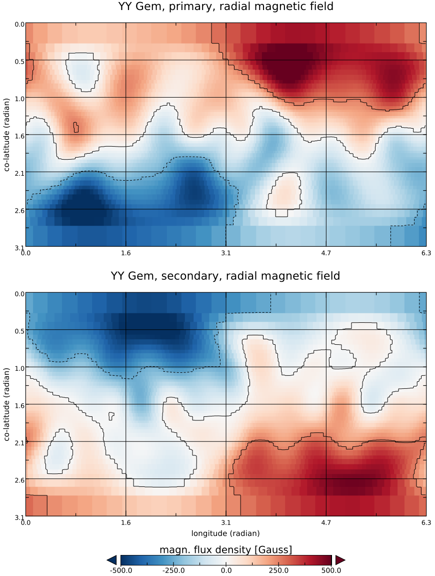
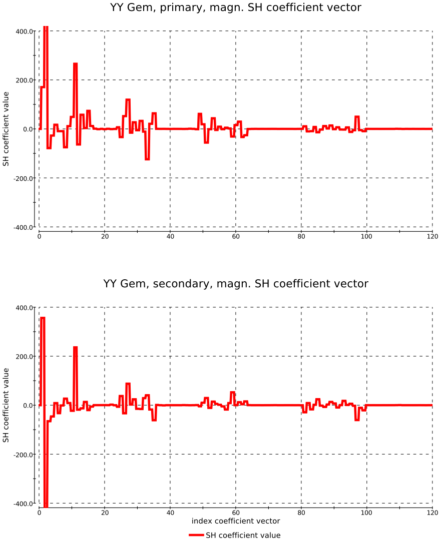
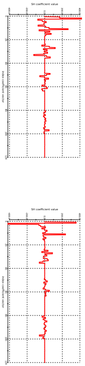


⇒ spherical harmonic analyses ⇒



Example: YY Gem

Linear system of equations ($R_0 = 4R_\odot$)



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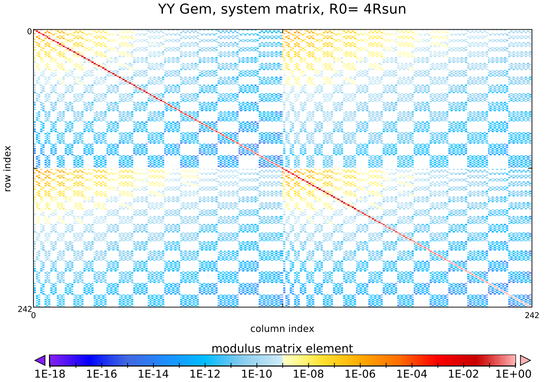
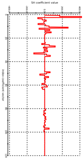
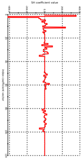


$$\cdot \begin{pmatrix} \mathbf{c}^{(1)} \\ \mathbf{c}^{(2)} \end{pmatrix}$$

Example: YY Gem

Linear system of equations ($R_0 \rightarrow \infty$)





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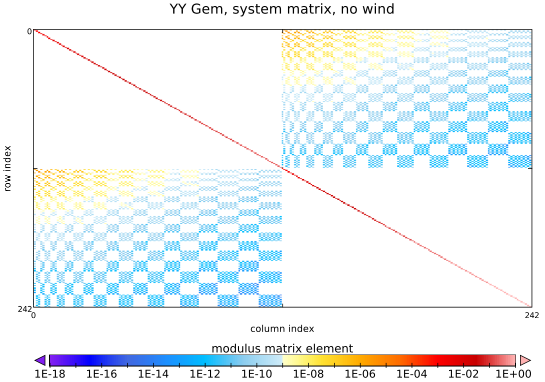

$$\cdot \begin{pmatrix} \mathbf{c}^{(1)} \\ \mathbf{c}^{(2)} \end{pmatrix}$$

Example: YY Gem

Joint magnetosphere – source surface radius $R_0 >> d_{\mathrm{YYGem}}$