The uniqueness problem of ZDI with Stokes V spectra

lessons from \( \tau \) Sco inversions


Oleg Kochukhov

oleg.kochukhov@physics.uu.se

Uppsala University, Sweden

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Motivation

- τ Sco: bright, slowly rotating, benchmark B0V star
- Weak and unusually complex magnetic field (Donati et al. 2006)
- Provides important observational constraints for fossil field theory
- Variability of the magnetospheric X-ray emission is not compatible with extrapolation from the surface ZDI maps (Ignace et al. 2010)
Goals

◆ Perform an independent ZDI analysis of τ Sco based on archival Stokes V data

◆ Assess reliability of ZDI inversions and role of Stokes QU observations

◆ Explore sensitivity of inversions to different forms of magnetic field parameterisation

◆ Address the question of self-consistency between ZDI and potential field extrapolation
Methodology

◆ InversLSD ZDI code (Kochukhov et al. 2014)
◆ 49 ESPaDOnS Stokes V observations (Donati & Landstreet 2009 + MiMeS), LSD
◆ Local Stokes profiles approximated with a Milne-Eddington analytical solution
◆ Field parameterizations
  – Direct
  – General harmonic ($\alpha \neq \beta, \gamma \neq 0$)
  – Restricted harmonic
    (a (a $\neq \beta, \gamma = 0$ and $\alpha = \beta, \gamma \neq 0$)

\[
B_{r}(\theta, \phi) = - \sum_{\ell=1}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi),
\]

\[
B_{\theta}(\theta, \phi) = - \sum_{\ell=1}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \left[ \beta_{\ell,m} Z_{\ell,m}(\theta, \phi) + \gamma_{\ell,m} X_{\ell,m}(\theta, \phi) \right],
\]

\[
B_{\phi}(\theta, \phi) = - \sum_{\ell=1}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \left[ \beta_{\ell,m} X_{\ell,m}(\theta, \phi) - \gamma_{\ell,m} Z_{\ell,m}(\theta, \phi) \right].
\]
Comparison of ZDI codes

Donati & Landstreet (2009)

Kochukhov & Wade (2016)
Harmonic vs. direct ZDI

\[ \Lambda \sum_i (B_r^i - B_r^{i+1})^2 \]

\[ \Lambda \sum_{\ell} \ell^2 (\alpha_{\ell,m}^2 + \beta_{\ell,m}^2 + \gamma_{\ell,m}^2) \]
Is there really a toroidal field?

\[ \alpha \neq \beta, \gamma = 0 \]
Linked poloidal field components

$\alpha=\beta, \gamma\neq0$

$\alpha\neq\beta, \gamma\neq0$
Summary for different parameterisations

<table>
<thead>
<tr>
<th>Parameterisation of magnetic maps</th>
<th>$\sigma_Y \times 10^{-5}$</th>
<th>RMS field (G)</th>
<th>Magnetic energy</th>
<th>Total</th>
<th>pol:tor</th>
<th>$\ell &lt; m/2$</th>
<th>$\ell \geq m/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>direct</td>
<td>7.9</td>
<td>168 142 211</td>
<td>1.18</td>
<td>–</td>
<td>–</td>
<td></td>
<td></td>
</tr>
<tr>
<td>harmonic, $\beta_{\ell,m} \neq \alpha_{\ell,m}, \gamma_{\ell,m} \neq 0$</td>
<td>8.1</td>
<td>160 118 215</td>
<td>1.09</td>
<td>45.6:54.4</td>
<td>25.6:74.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>harmonic, $\beta_{\ell,m} \neq \alpha_{\ell,m}, \gamma_{\ell,m} = 0$</td>
<td>8.2</td>
<td>356 206 223</td>
<td>2.76</td>
<td>100.0:0.0</td>
<td>34.3:65.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>harmonic, $\beta_{\ell,m} \neq \gamma_{\ell,m} \neq 0$</td>
<td>8.4</td>
<td>663 900 598</td>
<td>20.34</td>
<td>48.6:51.4</td>
<td>6.3:93.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $\alpha \neq \beta$ may be preferred on mathematical grounds (simplicity, regularisation)
- $\alpha = \beta$ is preferred on physical grounds (matching to exterior field)

$$
\beta_{\ell,m} = \frac{(\ell + 1)(R_s/R_\star)^{2\ell+1} - \ell - 1}{(\ell + 1)(R_s/R_\star)^{2\ell+1} + \ell}
$$

$\alpha = \beta$ is preferred on physical grounds (matching to exterior field)
Theoretical MHD models

Fossil field relaxation simulations (Braithwaite et al.)
Potential field extrapolation

\(\alpha \neq \beta, \gamma \neq 0\)

\(\alpha = \beta, \gamma \neq 0\)

\(\alpha \neq \beta, \gamma = 0\)

ZDI + PFSS

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Breaking degeneracies

◆ Faster rotation helps a bit
Breaking degeneracies

◆ Faster rotation helps a bit
◆ Stokes $QU$ profiles is an ultimate solution

\[ \alpha \neq \beta \quad \gamma \neq 0 \]

\[ \alpha \neq \beta \quad \gamma = 0 \]

\[ \alpha = \beta \quad \gamma \neq 0 \]
Conclusions

◆ Good agreement of independent ZDI inversions if the same field parameterisation is adopted

◆ Different harmonic field parameterisations yield significantly different ZDI solutions, which fit the same Stokes V data

◆ “Unlinking” of radial and horizontal poloidal field components is an important (but physically unjustified) degree of freedom

◆ PFSS extrapolation can be done consistently with ZDI, resulting in a different surface field topology

◆ Non-uniqueness can be avoided by incorporating linear polarisation information in ZDI inversions. ESPaDOnS Stokes QU of τ Sco will take place in 2016B
Cool Stars 19
Uppsala, June 6-10

Plenary session:
Magnetic activity and its impact on planets

Splinter sessions:
Surface magnetism
Solar & stellar activity and variability